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The optimal configuration of the multistage model with embedded chance constraints of mini-type hydroelectric-photovoltaic power distribution networks on the basis of the improved genetic algorithm

Zhu Ziwei[1]*, Liu Zhuorui[1]

Abstract. The grid-connection of mini-type hydroelectric, photovoltaic and other renewable energy sources has brought remarkable volatility and randomness to the system of power distribution network. But during the planning of that system, the comprehensive model of hydroelectric-photovoltaic planning model is rarely considered. This article brings forward multistage planning model with embedded chance constraints to address the issue of comprehensive planning of mini-type hydroelectric photovoltaic power distribution networks. Firstly, the multistage within different periods of time model which considers the sequence of the hydroelectric generator, the photovoltaic power generator and the capacitors was established. The objective functions of the three stages were respectively set as minimizing annual cost, minimizing transmission losses and abandoned water, and minimizing the times of the switching of transformer taps as well as of the switching of capacitors. Secondly, the objective function, the chance constraint of state variable and control variable and the constraint of the active and reactive power output of the hydroelectric generator, the photovoltaic power generator and the capacitors was comprehensively considered. After that, crossover model was introduced, in order to rectify the prematurity of genetic algorithm. Lastly, the form of IEEE 33-bus system can effectively enhance the utility of resources, reduce transmission losses and gross cost. The comprehensive model was also capable of keeping the voltage of the network stable and lower the times of adjustment of the transformers and the capacitors, improve the durable years of the facility.

Key words. Time interval multi state model, chance constraint, multistage programming model, cross model, improved genetic algorithm.

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[1] Information Engineering School of Nanchang University, Nanchang, Jiangxi, 330031, China.
*Corresponding Author, E-Mail: `268836778@qq.com`

1. Introduction

With the massive use of fossil energy around the globe, the problem of environment and energy has been increasingly severe, and nations are paying more and more emphasis on the study of renewable resources [1]. The united operation of the mini-type hydroelectric-photovoltaic power station has an outstanding character of complementation [2, 3]. The installation of mini-type hydroelectric-photovoltaic power generation has converted the traditional active power distribution into active network that consist of distributed power supply. And this change has a remarkable influence on the transmission losses, voltage quality and stability. reference [4] brought forward a multimarket model of locating and sizing that considered multiple scenarios as well as time sequence in order to reduce the transmission losses and outage costs; Reference [5] apart from the consideration of time sequence, further considered the environmental benefits of the locating, sizing and types the distributed power supply. Regarding the uncertainty of power supplied by new energies, reference [6] conducted chance constraints on voltage, which constructed a model of reactive power optimization that considers the random supply of photovoltaic power station in order to minimize the transmission losses. The above-mentioned methods were all quite effective, but at present, there's still a vacancy regarding the comprehensive planning models of both the active and reactive power which target the mini-type hydroelectric-photovoltaic power distribution network [7]. This article adopted a multistage planning model with embedded chance constraints. That model is in essence, a nonlinear mixed-integer optimization problem which includes both discrete variables and continuous variables. Reference [8] adopted an improved version of genetic algorithm on the basis of an interlace model. That algorithm can divide the population according to the similarity measure of different chromosome. After that, survival competitions will be conducted among the offspring, and greedy algorithm will be used to sift out the superior individuals. In this way, not only prematurity can be avoided and diversity guaranteed, the rate of convergence can also be assured. Lastly, this article verified the effectiveness of the multistage planning model and the genetic algorithm by the simulation generated by IEEE33 node system

2. The sequential multi-state model of random variables

2.1. The probabilistic model of hydroelectric, photovoltaic electric and load

The active output of hydroelectric generator is in direct proportion to its water head and outflow. the active output of the hydroelectric generator can be approximately described as:

$$P = 9.81\eta Q h\,, \tag{1}$$

within the equation (1): η represents the generating efficiency of mini-type hydroelectric generator; Q represents the rate of flow passing the water turbine.

$$f(Q) = \frac{\beta^{\alpha}}{\Gamma(\alpha)}(Q_0 - a_0)^{\alpha-1} e^{-\beta(x-a_0)} \,. \tag{2}$$

Within the equation (2): Γ is a Gamma function, α, β, a_0 are the distribution shape measure and the location parameter of the Pearson III distribution:

$$f(P_M) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\cdot\Gamma(\beta)} \cdot \left(\frac{P_M}{P_{\max}}\right)^{\alpha-1} \cdot \left(1 - \frac{P_M}{P_{\max}}\right)^{\beta-1} \,, \tag{3}$$

Within equation (3), a, β can be described in reference[2]; $P_{\max} = A\eta r_{\max}$ refers to the maximum power output of the photovoltaic array, $r_{\max}$ refers to the maximum light intensity;.The load can be approximately stimulated by using normal distribution, μ refers to mathematical deviation. and its probability density function is:

$$f(p) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{(p-\mu)^2}{2\sigma^2}\right] , \tag{4}$$

2.2. Sequential multi-state models of hydroelectric, photovoltaic electric and load

The sequential multi-state model of the active and reactive power output of mini-type hydroelectric generator is to divide the conditions of water flow into several categories, and hereby[9]. M is taken as the number of categories. The probability of active power output being within a certain condition can be expressed as:

$$F(i) = \int_{p_{(i-1)}}^{P_i} f(x)\cdot dx \qquad i = 1, 2, \ldots, M \,, \tag{5}$$

The probability of active power output being within a certain condition can be expressed as:

$$F_m(i) = \int_{p_{m(i-1)}}^{P_{mi}} f(x)\cdot dx \qquad i = 1, 2, \ldots, N \,, \tag{6}$$

The probability of each segment can be expressed as:

$$F_W(i) = \int_{\frac{i-1}{W}(P_{\max}-P_{\min})}^{\frac{i}{W}(P_{\max}-P_{\min})} f(P)dP \qquad i = 1, 2, \ldots, W \,, \tag{7}$$

Within equation (7): W is taken as the number of segments. Refers to the status value of the middle point of segment i. M, N, W referred to in this article respectively take the value of 3, 3, 5.

3. Multistage planning with embedded chance constraints of mini-type hydroelectric-photovoltaic power distribution network

3.1. Model of the planning of multistage chance constraints

The first-stage plan's objective function is to minimize the annual cost by referring to the model of reference [10]. The model shall be modified as:

1) Objective function: $\min \overline{F_1}$

2) Constraint conditions:

a) The probability constraint of the objective function:

$$P\left\{f(\mathbf{x},\varepsilon) \le \overline{F_1}\right\} \ge \alpha, \tag{8}$$

b) The probability constraint of the branches' power:

$$P\left\{P_l(\mathbf{x},\varepsilon) \le P_l^{\max}\right\} \ge \beta_P, i \in \Omega_{line}, \tag{9}$$

c) The probability constraint of the node voltage

$$P\left\{U_i^{\min} \le U_i(\mathbf{x},\varepsilon) \le U_i^{\max}\right\} \ge \beta_U, i \in \Omega_{node}, \tag{10}$$

d) The constraint of the installed capacity:

$$\begin{cases} \sum\limits_{n=1}^{N} S_{ci} \le \rho P_{\max} \\ 0 \le S_{pvi} \le S_{pvi}^{\max} \\ 0 \le S_{ci} \le S_{ci}^{\max} \\ 0 \le S_{wi} \le S_{wi}^{\max} \end{cases}, \tag{11}$$

e) The constraint of the power balance:

$$\begin{cases} P_i = U_i \sum\limits_{j=1}^{N} U_j (G_{ij}\cos\theta_{ij} + B_{ij}\sin\theta_{ij}) \\ Q_i = U_i \sum\limits_{j=1}^{N} U_j (G_{ij}\sin\theta_{ij} - B_{ij}\cos\theta_{ij}) \end{cases}, \tag{12}$$

The second-stage plan is to establish the objective function aiming at minimizing the transmission losses .The model shall be modified as:

1) The objective function: $\min \overline{F_2}$

2) The constraint conditions:

a) The probability constraint on the objective function:

$$P\left\{f(\mathbf{x},\varepsilon) \le \overline{F_2}\right\} \ge \alpha, \tag{13}$$

b) The power flow constraint on the power distribution network:

$$\begin{cases} P_{Gi}^{t} = P_{Li}^{t} - U_{i}^{t} \sum\limits_{j=1}^{N} U_{j}^{t} (G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}) \\ Q_{Gi}^{t} = Q_{Li}^{t} - U_{i}^{t} \sum\limits_{j=1}^{N} U_{j}^{t} (G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij}) \end{cases}, \tag{14}$$

c) The probability constraint on the state variables:

$$P \left\{ U_{i}^{\min} \leq U_{i}^{t} \leq U_{i}^{\max} \right\} \geq \beta, \tag{15}$$

d) The constraint on control variables:

$$\begin{cases} 0 \leq P_{Wi}^{t} \leq S_{Wi} \\ Q_{in}^{pv,t} \leq Q_{in,\max}^{pv} \\ Q_{ab}^{pv,t} \leq Q_{ab,\max}^{pv} \\ -0.9 \leq \cos \varphi \leq 0.9 \end{cases}, \tag{16}$$

The third-stage plan aims at lowering the number of times of the switching of the transformer taps and the switching of capacitors, The model shall be modified as:

1) The objective function

$$\min f = \min \sum_{h=1}^{24} \left[(C_{tap} \left| b_{tap}^{h} - b_{tap}^{h-1} \right| + C_{k} \left| k_{c,n}^{h} - k_{c,n}^{h-1} \right|) \right], \tag{17}$$

2) The constraint conditions:

a) The constraints on the upper and lower bounds of the transformer taps

$$b_{tap,\min} \leq b_{tap}^{h} \leq b_{tap,\max} \tag{18}$$

b) The constraint on the number of times of the switching of the transformer and the capacitor

$$\sum_{h=1}^{24} \left| b_{tap}^{h} - b_{tap}^{h-1} \right| \leq A_{tap}, \tag{19}$$

$$\sum_{h=1}^{24} \left| k_{c,m}^{h} - k_{c,m}^{h-1} \right| \leq A_{c,m}, \tag{20}$$

Within these equations: $\overline{F_1}$, $\overline{F_2}$, are the minimum value of $f(\mathbf{x}, \varepsilon)$ when the probability level is at least at α; K_p is the penalty coefficient of the quantity of abandoned water; P_{W0i} and P_{Wi} are respectively the rated generated output of the hydroelectric generator; P_{Gi}^{t} and Q_{Gi}^{t} are respectively the active and reactive power output of the power supply at time t; P_{Li}^{t} and Q_{Li}^{t} are respectively the active and reactive power load of node i at time t; $Q_{in}^{pv,t}$ and $Q_{ab}^{pv,t}$ are respectively the reactive power output generated and absorbed by the photovoltaic power generator at time t; C_{tap} and C_k are respectively the loss caused when every time the transformer taps and the capacitors are switched; b_{tap}^h, $k_{c,n}^h b_{tap}^h$ and $k_{c,n}^h$ are respectively the gear stage the transformer taps are at and the number of capacitor sets that are put into operation; A_{tap} is the maximum number of times of the transformer taps' switching within a day; $A_{c,m}$ is the maximum number of times of switching the capacitor set No. m can take within a day. Others parameters can be described in reference[11].

4. The solution process of multistage planning model on the basis of the improved genetic algorithm

The multistage planning model with embedded chance constraints established above is in essence, a nonlinear mixed-integer optimization problem which includes both discrete variables (such as the number of times of the switching of capacitors, the location of transformer taps, the location and capacity of the hydroelectric generator, of the photovoltaic power generator and of the capacitors)and continuous variables(such as the active and reactive power output of the hydroelectric generator and of the photovoltaic power generator) algorithm possesses great robustness. It has obvious superiority within the area of soling complicated problems of optimization. By referring to the idea of reference [12], this article puts forward the genetic algorithm of interlace model.

4.1. Improve the coding of the genetic algorithm

a) The code of the first-stage control variable:

$$\mathbf{X} = [S_{w1}, \ldots, S_{wi}, S_{pv1}, \ldots, S_{pvi}, S_{c1}, \ldots S_{ci}] \,, \tag{21}$$

Within the equation: S_{wi}, S_{pvi} and S_{ci} are respectively the number of the candidate positions for the hydroelectric generator, the photovoltaic power generator and the capacitors. The value taken represents the number of the facilities that are switched on. If the value is taken as 0 it means no facility is switched on.

b) The code of the second-stage control variable:

$$\begin{aligned}\mathbf{X}_{mnt} =&[Q_{pv10}, Q_{pv11}, \ldots, Q_{pvmt} P_{w10}, P_{w11}, \ldots, \\ &P_{wnt}, Q_{w10}, Q_{w11}, \ldots, Q_{wnt}] \,,\end{aligned} \tag{22}$$

The value taken represents the number of the facilities that are switched on. If the value is taken as 0 it means no facility is switched on.

c) The code of the third-stage control variable:

$$\mathbf{X}_{ij} = [b_{ij,tap1}^h, b_{ij,tap2}^h, \ldots, k_{ij,c,1}^h, k_{ij,c,2}^h, \ldots] \,. \tag{23}$$

The value taken represents the number of the facilities that are switched on. If the value is taken as 0 it means no facility is switched on.

4.2. Procedures for Improving The Genetic Algorithm

1) Input the initial network data and establish the multistage model of the hydroelectric generator, the photovoltaic power generator and the load at different moments; 2) encode the control variables of the three stages; 3) Generate the initial population of the first-stage plan's control variable, iteration is set as 1; 4) Generate initial population of the second-stage plan regarding every chromosome of the first-stage; 5) Establish adjacent matrix D by calculating the Euclidean distance between any two random chromosomes of the initial population of the second-stage; 6) Calculate the minimum spanning tree of the adjacent matrix D in step 5 by Prim Algorithm; 7) Calculate the average $\overline{W}$ of T, and calculate the Maximum weight that is less than $\delta \times \overline{W}$ under the threshold value V of T. δ's range is (0, 1). Then traverse T, search for all the boundaries that are bigger than threshold v. After that, disconnect it, number the sub-connection diagrams obtained as C_N; 8) Apply crossover probability gambling selection method to select a chromosome from the second stage to be crossed, and number that chromosome as C_i, select the best chromosome within the same group as the other chromosome X_j to be crossed. The crossover of these two chromosomes are regarded as the operation within the same population. After that, choose C_j which is the most remote one from C_i and select the chromosome it. Then, conduct interlace operation towards X_i and X_k choose the better individual from the two modes as the descendant, which should hereby be referred to also as the optimal individual of tage 2; 9) After obtaining the result of stage2 and the initial result of stage 1, ascertain the initial population of stage3, reapply step5-8, replace the chromosome of stage2 with the chromosome of the initial population of stage 3. In this way, the optimal individual of stage3 can be obtained; 10) Apply the first-stage operation to the optimal individuals of stage2 and stage3, calculate the objective function of each one these 2 chromosomes, and verify the confidence coefficient level of the objective functions and the constraint conditions; 11) Continue to reapply step5-8, replace the chromosome of stage2 within with the initial population of stage1, and generate a new planning population;12) Conduct the end condition judgment, if the generation of inheritance is greater than the set generation, the calculation can be ended. If otherwise, return to step5.

5. Case study

5.1. Introduction

This article applies the simulated analysis of IEEE 33 node power distribution network system to conduct amendment. Add on-load tap changing transformers at node1 and node12, the ratio of transformation is $0.9 \sim 1.1$, the top and bottom tap positions are ±8, $0.9 \sim 1.1$, the stepped size is 1.25%; the load level is 4015kW ;the positions of the to-be-installed hydroelectric generators are 8,20,31; The positions of the installment of the photovoltaic power generator are 6,13,23,29, The to-beselected positions of the capacitors are 3,7,11,19,24,26. The initial investment is, The maintenance charge of the facilities is , the running cost of electricity per unit is ,the maximum permeability is ,the unit price of electricity, ,and other relevant parameters can be obtained from reference[4,13]; The rated capacity of the small hydroelectric generator and the photovoltaic power generator is 300kW;The capacity of the reactive power compensation per unit is 10kvar, with 10 units in total, The maximum number of times of switching is 4; and are both 400 Yuan per time; the precision of power flow, $\varepsilon 1$ = 10-6; the scale of the genetic algorithm's population, N, takes the value of 100, the presupposed number of generations is 100, the crossover probability PC, is 0.8, the mutation probability Pm is 0.02, the clustering coefficient is 0.99.

5.2. The plan and the results

According to the models the solutions mentioned above, the following 7 scenarios will by analyzed:Scenario1: Firstly, the hydroelectric generators are planned. Secondly, the photovoltaic generators are planned. Lastly, the capacitors are planned. Scenario2: Firstly, the photovoltaic generators are planned. Secondly, the hydroelectric generators are planned. Lastly, the capacitors are planned. Scenario3: Firstly, the hydroelectric generators are planned. Secondly, the capacitors are planned. Lastly, the photovoltaic generators are planned. Scenario4: Firstly, the photovoltaic generators are planned. Secondly, the capacitors are planned. Lastly, the hydroelectric generators are planned. Scenario5: Firstly, the capacitors are planned. Secondly, the hydroelectric generators are planned. Lastly, the photovoltaic generators are planned. Scenario6: Firstly, the capacitors are planned. Secondly, the photovoltaic generators are planned. Lastly, the hydroelectric generators are planned.Scenario7: Comprehensively plan the hydroelectric generator, the photovoltaic power generator and the capacitorsThe results of the 7 scenarios were calculated, the results can be seen in table 1 and table 2. Table 1 indicates that the capacity of the parts planned first will be relatively big. In comparison, the parts that are planned later are relatively small. This is due to the fact that the index of the power network is not very good at first, after a round of planning, all the power network index will rise, which renders the insert of other massive supplement energies unnecessary.

By analyzing table 2, we can conclude that the result of the comprehensive planning of scenario7 can significantly reduce the cost of purchasing electricity from the major network. In another word, compared with the 6 former scenarios, the comprehensive planning is lower in transmission losses and has the best conglomerate benefit. Reference [14] uses only result of chance constraint. We can conclude from table 3 that, after taking the optimization of stage3 into account, the fluctuation of voltage reduced, and it needs less switching of both the capacitors and the transformers. This is because the traditional approach of voltage adjustment can only realize the discrete control of the voltage, it cannot deliver continuous control by adjusting the output power like the mini-type hydroelectric-photovoltaic power generator. The multistage optimization with embedded chance constraints can effectively coordinate the output of the hydroelectric generator and the photovoltaic.

It can also have smaller voltage under the condition that the voltage of every node is kept from the limiting value. Besides, it also has better stability.

Table 1. Results of Different Scenarios

scenarios	**Insertion position of photovoltaic**	**Inserted capacity of photovoltaic/ KVA**	**Insertion position of hydroelectric**	**Inserted capacity of hydroelectric/ KVA**	**Insertion position of capacitor**	**Inserted capacity of capacitors/ KVA**
scenario1	6,13,29	200300200	8,20,31	200300300	3,19,26	20,10,20
scenario2	6,13,23,29	200,300, 300300	20,31	200300	3,11,26	30,10,20
scenario3	6,13,29	200200200	8,20,31	200300300	3,11,19,26	30,20,10,20
scenario4	6,13,23,29	200,200, 300300	20,31	200300	3,11,19,26	30,20,30,20
scenario5	13,29	200300	20,31	300200	3,7,11, 19,24,26	20,30,30, 20,30,30
scenario6	6,29	300300	20,31	200200	3,7,11, 19,24,26	20,30,30, 20,30,30
scenario7	6,13,23,29	200,200, 300300	8,20,31	200200300	3,26	20,10

Table 2. Annual Charge for Use of Different Scenarios/ 10,000 Yuan

scenario	**Investment of facilities**	**Running cost**	**Maintenance charge**	**Electricity purchasing cost**	**All-in cost**
scenario1	79.24	18.9	30.92	622.35	751.41
scenario2	70.31	18.22	28.74	641.07	758.34
scenario3	73.61	18.73	29.81	663.69	785.84
scenario4	74.84	19.88	31.12	669.2	795.04
scenario5	63.35	17.86	27.23	711.43	819.87
scenario6	61.14	17.83	27.23	707.56	813.76
scenario7	83.76	19.73	31.27	609.32	741.08

Table 3. The Comparison Between Traditional Voltage Adjustment and The Multistage Adjustment with Embedded Chance Constraint

Method	**$\mathbf{Y}_{SSVF}$/KV**	**Times of switching**
		capacitor transformer
Traditional	1.922	3 7
multistage adjustment with embedded chance constraint	1.333	1 3

6. Conclusion

This article studied the comprehensive planning of mini-type hydroelectric-photovoltaic power station in order to address the considerable fluctuation and randomness that the grid-connection of renewable energies has brought to the power distribution network. This article established the sequential multistage planning model with embedded chance constraints for hydroelectric, photovoltaic and load. Applied the improved genetic algorithm in response to the existence of both the discrete variables and continuous variables within the process of solution for the model. According to the result of stimulation:1) Compared with the simple chance constraints models, the multistage planning model with embedded chance constraints has a higher probability of obtaining qualified voltage. This is because, firstly, the hydroelectric-photovoltaic power distribution network has better complementary character and stability compared with just the photovoltaic network, and secondly, the multistage planning model with embedded chance constraints can not only stimulate the actual planning model more effectively but also take more factors into consideration and thus has better all-round operation.2) The multistage comprehensive planning model has lower transmission losses, and better economic benefit compared with other separate sequential planning model.3) According to the calculation results of the multistage planning model with embedded chance constraints, the stability of voltage is better and also less switching is needed compared with the traditional approach,

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